International Journal of Mathematics and Computer Applications Research (IJMCAR)
ISSN(P): 2249-6955; ISSN(E): 2249-8060
Vol. 6, Issue 1, Feb 2016, 39-50

TRANS STELLAR •Journal Publications •Research Consultancy

SOLUTION OF SINGLE OBJECTIVE INVENTORY MODEL OF DETERIORATING ITEMS WITH FUZZY COST COMPONENTS AS TRIANGULAR FUZZY NUMBERS

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ABSTRACT

In this paper a multi item EOQ model with stock dependent demand for deteriorating items is considered in fuzzy environment. Different inventory costs and the amount of investment are represented as triangular fuzzy numbers The model has been solved by Robust ranking method, fuzzy optimization technique (FOT) and intuitionistic fuzzy optimization technique (IFOT). Nearest interval approximation method is used in FOT and IFOT. Pareto optimality test is done for fuzzy optimization and intuitionistic fuzzy optimization techniques. The methods are illustrated with a numerical example.

KEYWORDS: Fuzzy Inventory, Deteriorating Items, Nearest Interval Approximation, Robust's Ranking, Fuzzy Optimization, Intuitionistic Fuzzy Optimization

Received: Jan 06, 2016; **Accepted:** Jan 27, 2016; **Published:** Feb 01, 2016; **Paper Id.:** IJMCARFEB20165

Original Article

I. INTRODUCTION

Inventory problems are common in manufacturing, maintenance service and business operations in general. Often uncertainties may be associated with demand, various relevant costs. Generally demand rate is considered to be constant, time dependent, ramp type and selling price dependent. However in present competitive market stock-dependent demand plays an important role in increase its demand. Deterioration is one of the important factors in inventory system. Some items like food grains, vegetables, milk, eggs etc. deteriorating during their storage time and retailer suffers loss. In an inventory system available storage space, budget, number of orders etc. are always limited hence multi-item classical inventory models under these constraints have great importance.

In conventional inventory models, uncertainties are treated as randomness and are handled by probability theory. However in certain situations, uncertainties are due to fuzziness and in such cases the fuzzy set theory introduced by Zadeh [18] is applicable. Many researches introduced the concept of ranking function [] for fuzzy parameter which gives a representative value for it. Using ranking function fuzzy parameters can be easily changed into crisp one.

The fuzzy parameters changing into appropriate interval numbers are introduced by Gregorzwshi [5],Susovan chakrabortly et.al [15,16] proposed a method to solve an EOQ model using the concept of interval arithmetic. In fuzzy optimization the degree of acceptance of objectives and constraints are considered here. Now a days different modification and generalized form of fuzzy set theory have appeared intuitionistic fuzzy set (IFS). The concept of an IFS can be viewed as an alternative approach to define a fuzzy set in case where available information is not sufficient for the definition of an imprecise concept by means of a conventional fuzzy sets.

In this paper, a multi-item inventory model of deteriorating item with stock-dependent demand is formulated in crisp and fuzzy environment. Here the objective is to find an EOQ to maximize the profit. The problem is solved by

- Robust's ranking technique
- Fuzzy optimization technique
- Intuitionistic fuzzy optimization technique

The model is illustrated numerically and the results are obtained from different methods.

II.PRELIMINARIES

Definition (1): Atanassov [20], Let X={x_1 ,x_2,.......x_n} be a finite universal set. An Atanassov's intuitionistic fuzzy set (IFS) in a given universal set X is an expression $\hat{A}$ is given by $\hat{A} = \{\langle x_i, \mu_{\hat{A}}(x_i), \nu_{\hat{A}}(x_i)\rangle : x_i \in X\}$

Where the functions $\mu_{\hat{A}} : X \to [0,1]$ i.e. $x_i \in X \to \mu_{\hat{A}}(x_i) \in [0,1]$ and $\nu_{\hat{A}} : X \to [0,1]$ i.e. $x_i \in X \to \nu_{\hat{A}}(x_i) \in [0,1]$ define the degree of membership and the degree of non-membership respectively of an element $x_i \in X$ satisfy the condition: for every $x_i \in X$, $0 \le \mu_{\hat{A}}(x) + \nu_{\hat{A}}(x) \le 1$.

Definition (2): Let $\hat{A}$ and $\hat{B}$ be two Atanassov's IFSs in the set X. The intersection of $\hat{A}$ and $\hat{B}$ is defined as follows:

$$\hat{A} \cap \hat{B} = \left\{\left\langle x_i, \min\left(\mu_{\hat{A}}(x_i), \mu_{\hat{B}}(x_i)\right), \max\left(\nu_{\hat{A}}(x_i), \nu_{\hat{B}}(x_i)\right)\right\rangle \middle| x_i \in X\right\}.$$

Definition (3): Let $\Re$ be the set of all real numbers. An interval, moore[9], may be expressed as

$$\bar{a} = [a_L, a_R] = \{x : a_L \le x \le a_R, a_L \in \Re, a_R \in \Re\} \qquad \textbf{(1)}$$

Where a_L and a_R are called the lower and upper limits of the interval $\bar{a}$, respectively.

If $a_L = a_R$ then $\bar{a}$ = [a_L , a_R] is reduced to a real number a, where a= a_L= a_R. alternatively an interval $\bar{a}$ can be expressed in mean-width or center-radius form as $\bar{a} = \langle m(\bar{a}), w(\bar{a})\rangle$,where $m(\bar{a}) = \frac{1}{2}(a_L + a_R)$ and $w(\bar{a}) = \frac{1}{2}(a_R - a_L)$ are respectively the mid-point and half-width of the interval $\bar{a}$ .The set of all interval numbers in $\Re$ is denoted by I($\Re$).

Definition (4): (**Triangular Fuzzy Number**)

For a triangular fuzzy number A(x), it can be represented by A(a,b,c:1) with membership function $\mu_A(x)$ given by

$$\mu_A(x) = \begin{cases} \frac{(x-a)}{(b-a)} ; & a \le x \le b \\ 1 ; & x = b \\ \frac{(c-x)}{(c-b)} ; & b \le x \le c \\ 0 ; & otherwise \end{cases}$$

Definition (5): (α-cut of a Fuzzy Number)

The α-cut of a fuzzy number A(x) is defined as $A(\alpha)=\{x:\mu(x)\geq\alpha,\ \alpha\epsilon[0,1]\}$

Definition (6): Nearest Interval Approximation

According to Gregorzewski [5] we determine the interval approximation of a fuzzy number as: Let $\tilde{A}$ $=(a_1,a_2,a_3)$ be an arbitrary triangular fuzzy number with a α-cut $[A_L(\alpha),A_R(\alpha)]$

Then by nearest interval approximation method, the lower limit C_L and upper limit C_R of the interval are

$$C_L=\int_0^1 A_L(\alpha)d\alpha=\int_0^1\left[a_1+(a_2-a_1)\alpha\right]d\alpha=\frac{a_1+a_2}{2}$$

$$C_R=\int_0^1 A_R(\alpha)d\alpha=\int_0^1\left[a_3-(a_3-a_2)\alpha\right]d\alpha=\frac{a_2+a_3}{2} \quad \textbf{(2)}$$

Therefore, the interval number considering $\tilde{A}$ as triangular fuzzy number is $\left[\frac{a_1+a_2}{2},\frac{a_2+a_3}{2}\right]$.

Definition(7):Pareto-Optimality

$x^0\in X$ is said to be a Pareto optimal solution if there does not exist another $x\in X$ such that $f_k(x^0)\leq f_k(x)$ for all P = 1,2,......p and $f_j(x^0)<f_j(x)$ for at least one $j\in\{1,2,.......p\}$

III.ASSUMPTION

- The scheduling period is constant and no lead time.
- Replenishment rate is infinite.
- Selling price is known and constant.
- Demand rate is stock dependent.
- Shortages are not allowed.
- Deteriorating rate is age specific failure rate.

IV.NOTATIONS

T_i : Time period for each cycle for the i^{th} item.

R_i : Demand rate per unit time of i^{th} item. $[R_i = a_i+b_iq_i]$

θ_i :Deteriorating rate of i^{th} item.

$Q_i(t)$: inventory level at time t of i^{th} item.

C_H : Total Holding cost.

C_{1i} :Holding cost per unit of i^{th} item.

C_{3i} :Setup cost for i^{th} item.

S_{di} :Total deteriorating units of i^{th} item.

P_i :Selling price per unit of i^{th} item.

Q_i :Initial stock level of i^{th} item.

$PF(Q_i)$: Total profit of i^{th} item.

N :Number of items.

(wavy bar (~) represents the fuzzification of the parameters)

V. MATHEMATICAL FORMULATION

5.1 Crisp Model

As $Q_i(t)$ is the inventory level at time t of i^{th} item, then the differential equation describing the state of inventory is given by

$$\frac{d}{dt}Q_i(t) + \theta_i Q_i(t) = -(a_i + b_i Q_i(t)) \qquad 0 \le t \le T_i$$

Solving the above differential equation using boundary condition $Q_i(t)= Q_i$ at t=0, we get

$$Q_i(t) = -\frac{a_i}{(\theta_i + b_i)} + \left[Q_i + \frac{a_i}{(\theta_i + b_i)}\right] e^{-(\theta_i + b_i)t} \tag{3}$$

And using boundary condition $Q_i(t)= 0$ at $t=T_i$

$$\therefore T_i = \frac{1}{(\theta_i + b_i)} \log\left\{1 + \frac{(\theta_i + b_i)Q_i}{a_i}\right\} \tag{4}$$

The holding cost of i^{th} item in each cycle is

$$C_H = C_{1i}\, G_i\, (Q_i) \tag{5}$$

Where,

$$G_i(Q_i) = \int_0^{Q_i} \frac{q_i\, dq_i}{a_i + (\theta_i + b_i)q_i}$$

$$= \frac{Q_i}{(\theta_i + b_i)} + \frac{a_i}{(\theta_i + b_i)^2} \log\left\{1 + \frac{(\theta_i + b_i)Q_i}{a_i}\right\}$$

By neglecting the higher power terms, we get

$$G_i(Q_i) = \frac{Q_i^{\,2}}{2a_i}\left\{1 - 2\,\frac{(\theta_i + b_i)Q_i}{3a_i}\right\}$$

The total number of deteriorating units of the i^{th} item is

$$S_{di}(Q_i) = \theta_i G_i(Q_i)$$

The net revenue for the i[th] item is

$$N(Q_i) = (P_i - C_i)Q_i - P_i.S_{di}(Q_i)$$
$$N(Q_i) = (P_i - C_i)Q_i - P_i\theta_i.G_i(Q_i) \tag{6}$$

The profit of i[th] item is

$$PF(Q_i) = N(Q_i) - C_{1i}G_i(Q_i) - C_{3i}, \qquad \text{i=1,2,.....n.}$$

$$PF(Q_i) = (P_i - C_i)Q_i - P_i\theta_i.G_i(Q_i) - C_{1i}G_i(Q_i) - C_{3i} \qquad \text{i=1,2,.........n.} \tag{7}$$

$$PF(Q_i) = (P_i - C_i)Q_i - (C_{1i} + P_i\theta_i)G_i(Q_i) - C_{3i}, \text{ i=1,2,.............n.}$$

Hence the problem is

$$\text{Max } PF = \sum_{i=1}^{n}\left[(P_i - C_i)Q_i - (C_{1i} + P_i\theta_i)G_i(Q_i) - C_{3i}\right] \tag{8}$$

$Q_i \geq 0$, i=1,2,..........n.

5.2. Fuzzy Model

When above profit, cost, selling price, deteriorating rate and total budget become fuzzy, the said crisp model is transform to

$$M\tilde{a}x\ PF = \sum_{i=1}^{n}\left[(\tilde{P}_i - \tilde{C}_i)Q_i - (\tilde{C}_{1i} + \tilde{P}_i\tilde{\theta}_i)G_i(Q_i) - \tilde{C}_{3i}\right] \tag{9}$$

$Q_i \geq 0$, i=1,2,...............n

VI.MATHEMATICAL ANALYSIS

- **Robust Ranking Technique**

Given a convex fuzzy number $\tilde{a}$, the Robust's ranking index is defined by

$$R(\tilde{a}) = \int_0^1 0.5(a_\alpha^L, a_\alpha^U)\,d\alpha \quad ,$$

Where $\left(a_\alpha^L, a_\alpha^U\right)$ is the α-level cut of the fuzzy number $\tilde{a}$.

In this paper we use this method of ranking for the fuzzy numbers. The Robust's ranking index R($\tilde{a}$) gives the representative value of the fuzzy number $\tilde{a}$. Hence using this we can defuzzify into crisp one.

Robust's ranking technique [10,14] which satisfies compensation, linearity and additivity properties and provides results which are consistent with human intuition.

We apply Robust's ranking method[10,14] to defuzzify the fuzzy objective.

Taking P_i, i=1,2,.......n as fixed in (9) and Using this ranking function the fuzzy model (9) becomes,

$$Max\, PF^* = \sum_{i=1}^{n} \left[(P_i - R(\tilde{C}_i))Q_i - R(\tilde{C}_{1i})G_i(Q_i) - P_i R(\tilde{\theta}_i) G_i(Q_i) - R(\tilde{C}_{3i}) \right]$$

- **B. Fuzzy optimization Technique**

Using nearest interval approximation [5] to the objective and Robust's ranking [10,14] to the constant. The proposed model can be stated as

$$\max imize\{PF_L(Q_i), PF_R(Q_i)\} \quad \textbf{(10)}$$

Here the interval valued problem (10) is represented as

$$\max imize\{PF_L(Q_i), PF_C(Q_i) PF_R(Q_i)\} \quad PF_C = \frac{1}{2}[PF_L + PF_R] \quad \textbf{(11)}$$

To solve the above multi-objective problem (11) we have used the following fuzzy programming technique.

Step 1: Solve the multi-objective profit function as a single objective Profit function using one objective at a time and ignoring all others.

Step 2: From the results of step 1, determine the corresponding values for every objective at each solution derived.

Step 3: From step 2, obtain the lower bounds and upper bounds for each objective functions. Set U_k=max (PF_k) and L_k= min (PF_k) K=L,C,R.

Step 4: Define fuzzy linear membership function $\left(\mu_{PF_k}; k = L, C, R\right)$ for each objective function is defined by

$$\mu_{PF_k} = \begin{cases} 0 & ; PF_k \le L_k \\ \dfrac{PF_k - L_k}{d_k} & ; L_k \le PF_k \le U_k \\ 1 & ; PF_k \ge U_k \end{cases} \quad \textbf{(12)}$$

Step 5: After determining the linear membership function defined in(12) for each objective functions following the problem (11) can be formulated an equivalent crisp model

Max α,

$$\alpha \le \mu_{PF_k}(Q); k = L, C, R.,\ 0 \le \alpha \le 1, Q_i > 0$$

Step 6: Now the above problem can be solved by a non-linear optimization technique and optimal solution of α (say α*) is obtained.

Step 7: Now after obtaining α*, the decision maker selects the objective function, from among the objective functions PF_L, PF_C, PF_R. If he selects PF_C

Max PF_C ,

$$PF_L \le m_L, PF_C \le m_C, PF_R \le m_R \tag{13}$$

Where $m_L = U_L - \alpha * d_L; m_C = U_C - \alpha * d_C; m_R = U_R - \alpha * d_R;$

Step8: Pareto-optimal solution

Now after deriving the optimum decision variables, Pareto-optimality test is performed according to [16], let the decision vectors Q_i^* and the optimum values $PF_L^* = PF_L(Q^*),\ PF_C^* = PF_C(Q^*),\ PF_R^* = PF_R(Q^*),$ are obtained from (13). With these values, the following problem is solved using a non-linear optimization technique.

Subject to:

$$\left.\begin{array}{l}\min V = (\in_L + \in_C + \in_R) \\ PF_L + \in_L = PF_L^*, PF_C + \in_C = PF_C^*, PF_R + \in_R = PF_R^*, \\ \in_L, \in_C, \in_R \ge 0, Q \ge 0, \alpha \ge \beta \\ \alpha + \beta \le 1; \alpha, \beta \ge 0\end{array}\right\}$$

(14)

The optimal solution of (15), say $Q^{**}, PF_L^{**}, PF_C^{**}, PF_R^{**}$ are called strong Pareto Optimal solution provided V is very small otherwise it is called weak Pareto solution.

- **C. IF Programming Technique**

To solve multi-objective maximization problem given by (14), we have used the following IF programming technique.

For each of the objective functions $PF_L(Q), PF_C(\mathrm{Q}), \mathrm{PF}_R(Q)$, we first find the lower bounds L_L, L_C, L_R (best values) and the upper bounds U_L, U_C, U_R (worst values), where L_L, L_C, L_R are the aspired level achievement and U_L, U_C, U_R are the highest acceptable level achievement for the objectives $PF_L(Q), PF_C(\mathrm{Q}), \mathrm{PF}_R(Q)$ respectively and $d_k = U_k - L_k$ is the degradation allowance for objective $PF_k(Q)$, k=L,C,R. Once the aspiration levels and degradation allowance for each of the objective function has been specified, we formed a fuzzy model and then transform the fuzzy model into a crisp model. The steps of intuitionistic programming technique is given below.

Step 1: Solve the multi-objective profit function a single objective Profit function using one objective at a time and ignoring all others.

Step 2: From the results of step 1, determine the corresponding values for every objective at each solution derived.

Step 3: From step 2, obtain the lower bounds and upper bounds for each objective functions. Set U_k=max (PF_k) and L_k= min (PF_k) , K=L,C,R.

Step 4: Define membership function $\left(\mu_{PF_k}; k = L, C, R\right)$, and non membership function $\left(\nu_{PF_k}; k = L, C, R\right)$

for each objective function. A linear membership function is defined by

$$\mu_{PF_k} = \begin{cases} 1 & ; PF_K \le L_k \\ \left(\dfrac{PF_k - L_k}{d_k} \right) & ; L_k \le PF_k \le U_k \\ 0 & ; PF_K \ge U_K \end{cases} \tag{15}$$

A Linear non- membership function is defined by

$$\nu_{PF_k} = \begin{cases} 0 & ; PF_K \le L_k \\ 1 - \left(\dfrac{PF_k - L_k}{d_k} \right) & ; L_k \le PF_k \le U_k \\ 1 & ; PF_K \ge U_K \end{cases}$$

Step 5: After determining the membership and non-membership function defined in(14) for each objective functions following the problem (13) can be formulated an equivalent crisp model on the basis of definition 2 of this paper as

Max α , min β

$$\alpha \le \mu_{PF_k}(x) \quad ; k = L, C, R$$
$$\beta \ge \nu_{PF_k}(x) \quad ; k = L, C, R$$

$$\alpha \ge \beta \; ; \qquad \alpha + \beta \le 1; \alpha, \beta \ge 0$$

$$0 \le \alpha \le 1, Q_i > 0$$

Step 6: Now the above problem can be solved by a non-linear optimization technique and optimal solution of α (say α*) and β, (say β*) are obtained.

Step 7: Now after obtaining α* and β*, the decision maker selects the objective function, from among the objective functions PF_L, PF_C, PF_R. If he selects PF_C

$$\left. \begin{array}{l} Max\, PF_C \\ PF_L \le m_L, PF_C \le m_C, PF_R \le m_R \\ PF_L \ge n_L, PF_C \ge n_C, PF_R \ge n_R \\ Q \ge 0, \alpha \ge \beta; \\ \alpha + \beta \le 1; \alpha, \beta \ge 0 \end{array} \right\}$$

Where
$$\begin{array}{l} m_L = L_L + \alpha d_L; n_L = U_L - \beta d_L \\ m_C = L_C + \alpha d_C; n_C = U_C - \beta d_C \\ m_R = L_R + \alpha d_R; n_R = U_R - \beta d_R \end{array}$$

Step 8: Pareto-optimal solution

Now after deriving the optimum decision variables, Pareto-optimality test is performed according to [16], let the decision vector Q^* and the optimum values $PF_L^* = PF_L(Q^*),\ PF_C^* = PF_C(Q^*),\ PF_R^* = PF_R(Q^*),$ are obtained from (14). With these values, the following problem is solving using a non-linear optimization technique.

$$\left.\begin{array}{l}\min V = (\in_L + \in_C + \in_R) \\ \text{Subject to: } PF_L + \in_L = PF_L^*, PF_C + \in_C = PF_C^*, PF_R + \in_R = PF_R^*, \\ \in_L, \in_C, \in_R \geq 0, Q \geq 0, \alpha \geq \beta \\ \alpha + \beta \leq 1; \alpha, \beta \geq 0\end{array}\right\} \quad (16)$$

The optimal solution of (16), say $Q^{**}, PF_L^{**}, PF_C^{**}, PF_R^{**}$ are called strong Pareto Optimal solution provided V is very small otherwise it is called weak Pareto solution.

VII.NUMERICAL EXAMPLE

For all models let us assume

For n=2,

$P_1=10$, $C_1=7$, $a_1=110$, $b_1=0.5$, $\theta_1=0.025$, $P_2= 10$, $C_2=6.75$, $a_2=100$, $b_2=0.5$, $\theta_2=0.03$, $C_{11}=2$, $C_{12}=2.2$, $C_{31}=65$, $C_{32}=50$, B=1800.

- Taking $\tilde{C}_{11}$=(1.95,2,2.05); $\tilde{C}_{12}$=(2.15,2.20,2.25); $\tilde{C}_{31}$=(55,65,75); $\tilde{C}_{32}$=(40,50,60); $\tilde{\theta}_1$=(0.02,0.025,0.03) $\tilde{\theta}_2$=(0.025,0.03,0.035); $\tilde{C}_1$=(6.5,7,7.5); $\tilde{C}_2$=(6.25,6.75,7.25); $\tilde{B}$=(1700,1800,1900)

$$\max PF^* = 3Q_1 + 3.25Q_2 - 0.010226Q_1^2 - 0.0125Q_2^2 - 115$$

The optimum results are

Table 1

Max PF*	Q_1*	Q_2*
316.27738	146.68492	130

- Considering all values as same as case (A) and also taking $\tilde{P}_1$=(9.5,10,11); $\tilde{P}_2$=(9.75, 10,11);.

Table 2

	PF_L	PF_C	PF_R
PF_L	184.67881	162.78795	90.677814
PF_C	342.71953	363.87741	339.55730
PF_R	500.75962	564.96589	588.43534

The optimum value of α is **0.75002106**

Optimum results when PF_L is chosen as the most important objective function

Table 3

PF_L*	PF_C*	PF_R*	Q_1*	Q_2*
161.18053	357.79789	554.4140663	132.77717	152.94231

Pareto Optimal Solution

Table 4

V	PF_L**	PF_C**	PF_R**	Q_1**	Q_2**
0.70661E-05	161.18052	357.79788	554.41407	132.77717	152.94232

- Considering all values as same as case (B) and also taking $\tilde{P}_1$=(9.5,10,11); $\tilde{P}_2$=(9.75, 10,11);.

The optimum value of α and β

Table 5

α	β
0.75002106	0.24997894

Optimum results when PF_L is chosen as the most important objective function

Table 6

PF_L*	PF_C*	PF_R*	Q_1*	Q_2*
161.18054	363.84990	566.5182	153.74993	140.95712

Pareto Optimal Solution

Table 7

V	PF_L**	PF_C**	PF_R**	Q_1**	Q_2**
0.33021E-04	161.18053	363.84986	566.51819	153.71317	140.99336

VIII.CONCLUSIONS

In this paper, a real life inventory problem under the investment constraint environment has been proposed and solved by three different methods namely (i) Robust's ranking (ii) fuzzy optimization (FOT) (iii) intuitionistic fuzzy optimization (IFOT). Different inventory costs and the investment amount are considered as triangular fuzzy numbers nearest interval approximation method is used in FOT and IFOT. Linear membership function is taken for fuzzy objective. Robust's ranking method gives the less profit amount when compared with fuzzy optimization and intuitionistic fuzzy optimization. FOT and IFOT yields better results. Also strong Pareto optimality is obtained for FOT and IFOT.

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